Sem I (B)

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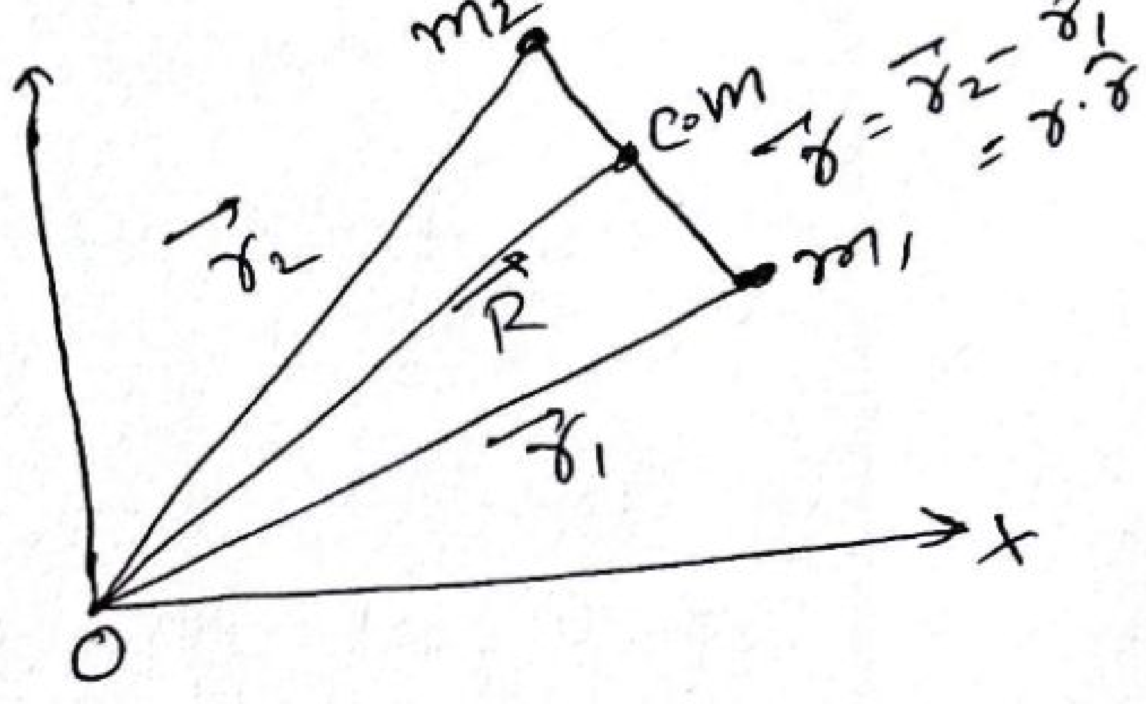
Q (a) What is reduced mass? Reduce two body problem to one body problem and obtain equation of motion for equivalent one body problem for two masses.

(b) What are the consequences of the motion of reduced mass when a large body attracts a very small body.

Ans (a) Reduced mass :→ Consider two mass points m_1 and m_2. If C.O.M for these two mass points and $\vec{R}$ be the radius vector for the C.O.M, then



$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

where $\vec{r}_1$ is the radius vector for mass point m_1

& r_2 that for mass point m_2

Equivalent one body Problem :→

Suppose there is no external force acting on the system and the only forces are those of mutual interaction, then the velocity of the C.O.M is constant. As the C.O.M must be on the line joining m_1 and m_2, the force on m_1 due to m_2 as well as the force on m_2 due to m_1 are both directed the C.O.M. Hence these forces are central.

If we consider the force on m_1 as $\vec{F}_{12} = F(r)\cdot\hat{r}$, the force on m_2 being equal and opposite will be denoted by

$$\vec{F}_{21} = -F(r)\cdot\hat{r}$$

$$\therefore m_1\frac{d^2\vec{r}_1}{dt^2} = \vec{F}_{12} = F(r)\cdot\hat{r} \quad \text{or} \quad \frac{d^2\vec{r}_1}{dt^2} = \frac{1}{m_1}F(r)\hat{r} \quad \text{------ (I)}$$

$$m_2\frac{d^2\vec{r}_2}{dt^2} = \vec{F}_{21} = -F(r)\hat{r} \quad \text{or} \quad \frac{d^2\vec{r}_2}{dt^2} = -\frac{1}{m_2}F(r)\hat{r} \quad \text{------ (II)}$$

Subtracting (II) from (I)

$$\frac{d^2\vec{r}_1}{dt^2} - \frac{d^2\vec{r}_2}{dt^2} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)F(r)\hat{r}$$

If we put $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} = \frac{m_1+m_2}{m_1 m_2}$ $\therefore \mu = \frac{m_1 m_2}{m_1+m_2}$

Then $\frac{d^2\vec{r}}{dt^2} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) F(r)\hat{r} = \frac{1}{\mu} F(r)\hat{r}$ or $\vec{F} = F(r)\hat{r}$ ... (iii)

Here 'μ' is known as reduced mass of the system and acts at a point known as centre of mass which divides the line joining the two masses in the inverse ratio of masses.

The relation $\mu\vec{r} = F(r)\hat{r}$ gives the eq^n of motion of a particle having mass equal to reduced mass μ at a vector distance $\vec{r}$ from one of the particles to the other and shows that two seperate eq^n of motion (i) & (ii) have reduced single mass

(b) Consequences of motion of reduced mass when a ~~single~~ large attracts a very small body :- when a large body of mass m_1 attracts a very small body of mass m_2, the motion of reduced mass has the consequences given below

(1) Reduced mass $\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{m_1 m_2}{m_1}$ $[\because m_2 << m_1)$

$\therefore \mu = m_2$

Therefore in such cases the reduced mass is practically equal to the mass of small body.

(2) If $\vec{r}_1$ and $\vec{r}_2$ are the respective distances of the bodies of mass m_1 and m_2 then

$m_1 r_1 = m_2 r_2$ or $r_1 = \frac{m_2}{m_1} r_2 = 0$

as m_2 is very small as compared to m_1, $\frac{m_2}{m_1} = 0$

Hence in such cases the C.O.M practically coincides with the centre of larger body.

In the absence of an external force, C.O.M of the system moves either with constant velocity or remains at rest. Therefore under the effect of the gravitational force between the two masses, only the motion of the lighter mass with respect to the heavier mass should be considered.