Radial and angular wave function for hydrogen atom:-

In quantum mechanics, the wave function of the hydrogen atom are described by solutions to the schrodinger equation is spherical coordinates. The wave function is characterised by the quantum numbers: principal quantum number n, azimuthal (or orbital angular momentum) quantum number l, and magnetic quantum number m.

The wave function can be separated into radial and angular parts:-

I. Radial wave function $R_{nl}(r)$

The radial part of the wave function depends on the distance r from the nucleus and is given by:-

$$R_{nl}(r) = A_{nl} \cdot \left(\frac{2Z}{na_0}\right)^{3/2} \cdot \left(\frac{r}{a_0}\right)^{l} \cdot e^{-Zr/na_0} \cdot L^{2l+1}_{n-l-1}\left(\frac{2Zr}{na_0}\right)$$

where, Z is atomic number (1 for H-atom)

a_0 = Bohr radius (approximately 0.529 Å)

$L^{2l+1}_{n-l-1}(x)$ are the associated Laguerre polynomials.

A_{nl} = normalization constant.

2. Angular wave function $Y_{lm}(\theta, \phi)$

The angular part is represented by spherical harmonics, given by:-

$$Y_{lm}(\theta, \phi) = N_{lm} \cdot P_l^m(\cos\theta) \cdot e^{im\phi}$$

where, N_{lm} is normalized constt.

P_l^m = Associated legendre polynomials

θ = polar angle, and ϕ is the azimuthal angle

Total wave function

The total wave function for the hydrogen atom can be expressed as the product of the radial and angular parts:-

$$\psi_{nlm}(r,\theta,\phi) = R_{nl}(r)\cdot Y_{lm}(\theta,\phi)$$

eg:- The ground state wave function ($n=1, l=0, m=0$) of the hydrogen atom would have:-

→ Radial part:-

$$R_{10}(r) = \frac{1}{\sqrt{\pi a_0^3}}\cdot e^{-r/a_0}$$

→ Angular part:-

$$Y_{00}(\theta,\phi) = \frac{1}{2\sqrt{\pi}}$$

Thus, total wave function

$$\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a_0^3}}\cdot e^{-r/a_0}\cdot\frac{1}{2\sqrt{\pi}}$$

This gives complete description of H-atom's wave function in the ground state.